A Cobb-Douglas Simulation. United States Manufacturing

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This paper produces an algorithm that reproduces the original results of Cobb and Douglas (1928) using restricted ordinary least squares. This algorithm is applied to the United States manufacturing industries during 1987-2023. Data is retrieved from the Bureau of Labor Statistics. A panel econometric model is implemented to analyze durable and nondurable goods with a granularity of three digits. Durable goods technology is more efficient than nondurables. At the industry level "other transportation equipment" is technologically more efficient, while "computers" is the most labor intensive, "chemicals" is the most capital intensive, and "motor vehicles" is the most input intensive.

Keywords: Cobb-Douglas, manufacturing, panel, simulation, United States

INTRODUCTION

The Cobb-Douglas production function was first used by the economist and senator Douglas and mathematician Charles W. Cobb, as the basis of their statistical research. They estimate the production function for the manufacturing sector in the United States for the period of 1899-1922 with annual data. Their estimation was a bold statistical attempt to find the shape of the production function, which was studied abstractly in economic theory, but that few had attempted to specify (Ikeo, 2022). Ikeo (2022) mentions that Solow uses the neoclassical assumption that factors are paid their marginal products or shares in output production, inaugurating then a new field called 'growth accounting' where economic theory and statistical estimation converge. Solow (1956) introduces the term of total factor productivity to designate technical change, while Cobb and Douglas (1928) use the term of technology for the same concept.

In their famous 1928 paper, Cobb and Douglas developed the idea of using a production function to relate an index of physical volume of manufactures in the United States to an index of the relative total capital in manufacturing, and an index of the probable average numbers of wage-earners employed in this sector. Considering this empirical setting, this paper employs an algorithm that uses Restricted Ordinary Least Squares (ROLS) to reproduce the empirical results of Cobb and Douglas (1928) as developed by Carbajal-De-Nova (2024). She found that her algorithm replicates the results of Cobb and Douglas exactly. These findings demonstrate the robustness of her method. In this paper, this algorithm is applied to the United States three digit manufacturing industries for the period 1987-2023, with annual data. The data is retrieved from the Bureau of Labor Statistics, and in specific the Office of Productivity and Technology. A panel model is used to elucidate the heterogeneity observed in the United States manufacturing sector at the industry granularity level.

The use of panel models to analyze economic phenomena has been a subject to critique. Griliches and Mairesse (1995) argue that the use of thinner and thinner slices of data, exacerbates estimation problems and misspecifications. Specifically, Mendershausen (1938) asserts that the production function is not "identifiable," because the input variables are determined simultaneously by the same forces that determines output. Thereof, the borders among exogenous and endogenous variables in the production functions are blurred. On the other hand, several studies highlight the advances of using panel models. For instance, Mairesse and Hall (1996) estimate a Cobb-Douglas production function using a panel model with the Generalized Method of Moments (GMM), addressing issues related to simultaneity and firm-specific effects. Their findings underscore the significance of this type of research in enhancing both French and United States manufacturing firms' performance. Similarly, Blundell and Bond (1999) focus on United States manufacturing firms to address challenges in estimating Cobb-Douglas production function using panel data, where GMM estimators and lagged instruments were used to yield unbiased estimators, which they use to reduce estimation problems like heterogeneity and simultaneity.

In Carbajal-De-Nova (2024), an algorithm is developed to reproduce the results of Cobb and Douglas (1928), as mentioned previously. This study compares the estimations of Cobb and Douglas (1928) with those obtained using the the algorithm based on ROLS. This comparison demonstrates that both sets of estimators are identical. In the present paper, this algorithm is applied to a panel model belonging to United States manufacturing industries. The analysis focuses on durable and nondurable goods at a three-digit level of granularity. These three digits represent 19 industries aggregate according to the North American Industry Classification System (NAICS) criteria. This panel econometric analysis is expected to provide estimates of technology, capital and labor shares in value added for type of good (see Table 2), and 19 industries (see Table 3), taking advantage of fixed and random effects. Cobb and Douglas (1928) use the following production function representation:

$$Y = A^{\eta} K^{\alpha} L^{\beta} \tag{1}$$

where Y is the product, A is technology, η represents the total factor productivity, K represents the capital production factor, L is the labor production factor, α is capital share in the product, β is labor share in the product. Cobb and Douglas (1928) transform equation (1) with logarithms, as follows:

$$logY = \eta logA + \alpha logK + \beta logL \tag{2}$$

Over time the Cobb Douglas production function as depicted on equation (1) has been widely used to measure changes in quantities of labor, and capital employed to produce a given volume of goods. Also, it has being used to measure the output shares of each production input or their marginal productivities. According with Griliches and Mairesse (1995), an innovation to the Cobb Douglas production function theoretical framework involves modelling profit as a stochastic variable, acknowledging the influence of random factors such as weather and unpredictable variations on factor performance. This acknowledgement allows to explain output entrepreneurial choice under a behavioral theoretical framework of decision-making. This innovation allows effects of output variance over entrepreneurial decisions. These authors mention that behavioral economics takes a stance in this theoretical setting, as maximization decisions are often based on expected rather than actual prices. This framework innovation assumes imperfect knowledge and inertia on the part of the entrepreneur, with inputs treated as independent of the production function disturbance term. Furthermore, as the use of the Cobb-Douglas production function has expanded various critiques have emerged. For instance,

a) It has been claimed that the Cobb-Douglas production function represents a static and purely accounting identity (Phelps and Brown, 1957),
b) The assumption of constant return to scale imposes a rigid functional form (Kmenta, 1967),
c) Key Cobb-Douglas production function assumptions like separability, substitution, perfect competition, homogeneity in inputs and outputs, perfect knowledge and aggregation have problems to hold empirically (Berndt and Christensen, 1973),
d) There is presence of multicollinearity, outliers, and absence of technical progress (Samuelson, 1979),

e) Simple Ordinary Least Squares estimates of the production function would be biased and inconsistent due to simultaneity and endogeneity issues (Griliches and Mairesse, 1995).
f) Certain production factors, such as “management” are difficult to quantify using standard economic indicators. Although, “management” is an important production factor, it frequently cannot be estimated (Nerlove, 1965).

This document is organized as follows: Section II presents the data used in this paper, including descriptive statistics. Section III contains the panel model and the methodology of the simulation algorithm. Section IV reports the econometric results. Finally, in the last section the conclusion is put forward.

DATA

The data used in the present investigation is retrieved from the Bureau of Labor Statistics, Office of Productivity and Technology (OPT). According to the OPT Handbook of Methods “sectoral output is defined as gross output less intra-industry transactions,” where “gross output is the total value of goods and services produced by an industry,” and “intermediate inputs are the foreign and domestically sourced goods and services used by an industry in the process of producing its gross output.” The OPT claims that “sectoral output and value-added output measures converge as the intermediate inputs produced and consumed within the sector approach the value of all intermediate input purchases.”

According to the OPT table titled “Annual total factor productivity and related measures for major industries,” sectoral output is equal to the sum of capital costs plus labor costs plus intermediate input costs, expressed in billions of current dollars. Cobb and Douglas (1928) use only capital and labor production factors in equation (1), where their total income sum will be equal to total valued added. Also, for these authors total value added would be equal to total physical product. Cobb and Douglas define capital as factor machinery, tools, equipment and factory building excluding raw materials, goods in process of manufacture and finished goods in warehouses. Given the OPT definition of sectoral output, the inclusion of intermediate input costs in the production function becomes a detachment of Cobb Douglas definition of value added or total physical product.

According with OPT definition of sectoral output, the inclusion of intermediate input costs could help in determining precisely manufacturing total factor productivity. In this context, intermediate input costs may help account for previously unmeasured production factors, potentially addressing gaps identified in Carbajal-De-Nova (2024). In the literature this gap often receives different names “total factor productivity”, “level of industry productivity”, “technical change”, “aggregate productivity growth”, “efficiency differences”, “measure of our ignorance”, or the “Solow residual” (Hall (1988), Griliches (1996), Olley and Pakes (1996), Basu et. al,. (2006), Del Gatto et. al,. (2011), Bartelsman et. al,. (2013)). The inclusion of intermediate inputs costs in the Cobb-Douglas production function estimation might mitigate bias from omitted production factors. Therefore, given OPT data availability, the original Cobb-Douglas functional form is modified to include the intermediate input costs as a production factor,

$$Y = f(K, L, I) \tag{3a}$$

This functional form is independent of measurement units. For example, consider this last equation in billions of current dollars:

$$Y * 1{,}000{,}000{,}000US\$ = f(K * 1{,}000{,}000{,}000US\$, L * 1{,}000{,}000{,}000US\$, I * 1{,}000{,}000{,}000US\$)$$

Collecting monetary terms in the left hand side, the billions of current dollars cancel out:

$$Y * 1{,}000{,}000{,}000US\$ = 1{,}000{,}000{,}000US\$ f(K, L, I)$$

$$\frac{Y * 1{,}000{,}000{,}000US\$}{1{,}000{,}000{,}000US\$} = f(K, L, I)$$

$$Y = f(K, L, I)$$

The same cancellation would have happened if billions of real dollars are instead considered. According to Nerlove (1965) the use of dollar values is an alternative measure of outputs and inputs in physical units. Therefore, it seems that the estimation of the Cobb-Douglas production function does not change with the measurement units. Next, Table 1 presents the descriptive statistic mean for United States manufacturing, durable and nondurable goods. This table also includes the mean for 19 industries at a three digit level granularity.

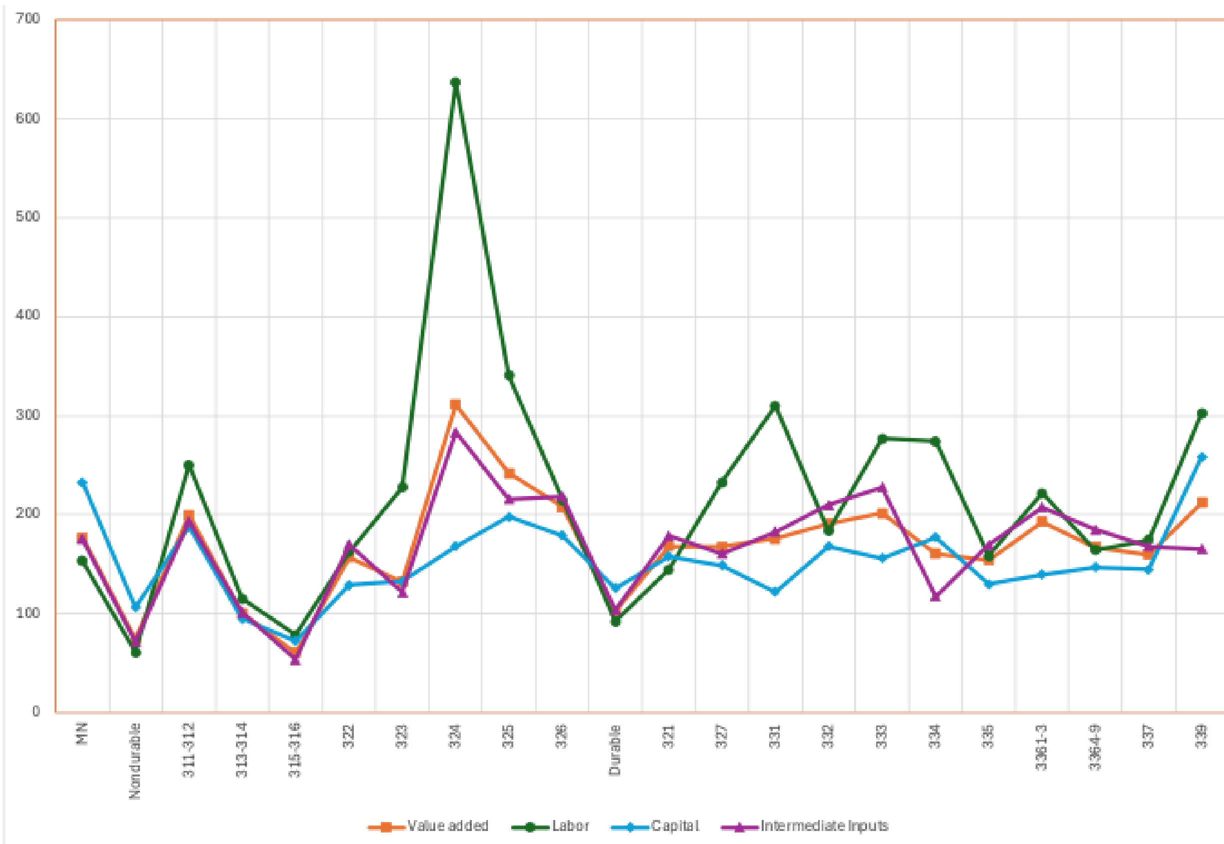
TABLE 1
UNITED STATES MANUFACTURING. MEAN: DURABLE AND NONDURABLE GOODS, AND THREE-DIGIT INDUSTRIES. BILLIONS OF CURRENT DOLLARS. 1987-2023

Naics code	Industry	Value added	Labor	Capital	Intermediate Inputs
MN	Manufacturing	176.96	153.20	232.68	175.27
	Nondurable goods	**74.26**	**61.13**	**106.66**	**71.45**
311-312	Food and beverage and tobacco products	199.55	250.12	187.59	193.46
313-314	Textile mills and textile product mills	100.55	115.13	95.02	101.34
315-316	Apparel and leather and applied products	60.52	78.59	72.47	53.52
322	Paper products	156.93	162.54	128.85	170.06
323	Printing and related support activities	132.14	227.66	132.76	121.55
324	Petroleum and coal products	311.25	636.48	168.19	283.24
325	Chemical products	242.16	340.7	197.79	215.79
326	Plastics and rubber products	207.97	215.33	178.90	218.73
	Durable goods	**102.69**	**92.06**	**126.01**	**103.81**
321	Wood products	168.23	144.79	157.90	178.68
327	Nonmetallic mineral products	167.41	232.85	148.87	160.69
331	Primary metal products	175.78	309.8	122.06	182.7
332	Fabricated metal products	191.12	183.60	168.01	209.7
333	Machinery	201.51	277.08	156.16	227.89
334	Computer and electronic products	160.68	274.33	177.23	117.45
335	Electrical equipment, appliances, and components	154.09	158.59	129.81	169.37
3361-3	Motor vehicles, bodies and trailers, and parts	193.00	221.80	139.47	207.66
3364-9	Other transportation equipment	167.06	164.3	146.94	185.09
337	Furniture and related products	160	174.67	144.52	167.95
339	Miscellaneous manufacturing	212.31	302.57	258.31	165.28

Notes: Durable and nondurable goods classification is taken from the Bureau of Economic Analysis, Table 6.4C. Labor are full-time and part-time employees by industry. Data for the 19 three digit industries are taken from the Bureau of Labor Statistics, Office of Productivity and Technology, Table Annual total factor productivity and related measures for major industries. NAICS stands for North American Industry Classification System code. The 336 industry is divided in two parts by the OPT. The Stata 18 program was used.

Next, Figure 1 visually represents the mean descriptive static reported on Table 1. The highest mean of value added, labor and intermediate inputs happen in the industry 324 Petroleum and coal products, which is a nondurable good. It can be observed that durable goods are also labor intensive since 331 Primary metal products and 339 Miscellaneous manufacturing expend more than 300 billion on this production factor.

FIGURE 1
UNITED STATES MANUFACTURING. MEAN: DURABLE AND NONDURABLE GOODS, AND THREE-DIGIT INDUSTRIES. BILLIONS OF CURRENT DOLLARS. 1987-2023



Labor cost, capital costs, intermediate input costs, and sectoral value added are reported in billions of current dollars in the OPT table mentioned above. In this paper, these time series are converted into index numbers using 1987 as base year. This conversion follows the procedure outlined by Cobb and Douglas (1928), for the generation of an index number with a base year corresponding to the first year of the analysis period.

PANEL MODEL

According to Griliches and Mairesse (1995), there is cross-sectional census data on manufacturing for the year of 1957, with states as units of observation. Nerlove (1965) discussed two approaches advanced by Klein (1953), each offering different economic interpretations of the stochastic elements involved on the Cobb Douglas production function. The first approach involves deriving a short-run industry supply function for a competitive industry using cross-sectional data from a sample of firms in the industry. The second approach develops a measure of relative economic efficiency based on estimates of production functions that differ from firm to firm. Nerlove (1965) further argues that, in models of production within regulated industries, the cost function is the most appropriate reduce-form equation for estimation purposes. This is because the assumption of a degree of returns to scale is invariant with respect to output level. Under this lens, the use of ROLS in the present analysis does not impose constant returns to scale to the manufacturing sector output, or any of its industries at three-digit aggregation level. Despites Carbajal-De Nova (1924) and Cobb and Douglas (1928) assume constant returns to scale, both authors find a technology estimate of 1.01. In this note, ROLS is expected to allow each manufacturing industry to express its corresponding technology estimate, even under the assumption of constant returns to scale.

Nerlove (1965) further argues that, in models of production within regulated industries the cost function is the most appropriate reduce-form equation for estimation purposes. This is because the assumption of a certain degree of returns to scale is invariant with respect to output level. Nerlove points out that the reduce-form possess certain desirable properties such as statistical consistency, and perhaps unbiasedness, as they are exactly identified. He also highlights the fundamental duality between cost and the production functions demonstrated by Shepard (1953). This demonstration guarantees a unique relationship between the empirically estimated cost function and the underlying production function. Here, the envelope theorem is mathematically justifying Shepard's lemma, based on how cost and production functions are related through optimization principles.

For their part, Mankiw, Romer and Weil (1992) assume an elasticity of substitution equal to one with two production inputs, implying unbiased technical change in their cross-country income differences econometric exercise. Here, the elasticity of substitution equals to one means the rate at which the input factors can be substituted for each other at a constant rate, equals the percentage change in the marginal rate of technical substitution to produce the same level of output. The transition of this model to incorporate input costs might be suitable, if the concerning information is available.

The modified functional form of the Cobb-Douglas functional panel model form is expressed on equation (3a) above. For convenience equation (3) is reproduced again in this section, now including an i industry specific subindex to account for individual industry effects:

$$Y_i = f(K_i, L_i, I_i) \tag{3b}$$

where Y_i is sectoral output for type of industry, K_i is capital costs for type of industry, L_i is labor costs for type of industry, I_i is intermediate input costs for type of industry, $i = MN, durable\ goods, nondurable\ goods, 311-312, 313-314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 3361-3363, 3364-3369, 337, 339$. In exponential form equation (3b) becomes equation (4):

$$Y_i = A_i^{\eta} K_i^{\alpha} L_i^{\beta} I_i^{\gamma} \tag{4}$$

In this equation, no production factors are omitted, since the sum of the variables exactly reproduces the output value, as discussed at the beginning of section II. Taking the logarithm of both sides of equation (4) gives:

$$logY_i = \eta logA_i + \alpha logK_i + \beta logL_i + \gamma logI_i \tag{5}$$

While Cobb and Douglas (1928) refer to η as technology, OPT refers to the same term as the manufacturing total factor productivity. As indicated above, these terms are equivalent in the context of the production function. Applying the logarithm function to the variables in the panel model allows the estimates to be interpreted as elasticity coefficients. These coefficients represent the percentage change in output resulting from a one percent change in each of the production factors, holding other factors constant.

The hypothesis of the panel model consists of considering the restriction that $\alpha + \beta + \gamma = 1$ embedded in the ROLS (Restricted Ordinary Least Square econometric methodology). ROLS does not impose any restriction on the estimate value of η, allowing it to express a measure of productivity efficiency. If $\eta = 1$, then the economy exhibits constant returns to scale, if $\eta > 1$, then the economy exhibits increasing returns to scale, if $\eta < 1$, then the economy exhibits decreasing returns to scale. As explained in the previous section, deflators are not used.

RESULTS

Table 2 reports the panel econometric estimates for the entire manufacturing sector without effects (baseline), as well as for durable and nondurable goods using fixed and random effects, all estimated with the simulation algorithm. Table 3 presents the results for an industry granularity of three-digits using ROLS.

TABLE 2
RESULTS OF THE ESTIMATES OF EQUATION (3a). UNITED STATES MANUFACTURING: DURABLES AND NONDURABLE GOODS. BASELINE, FIXED AND RANDOM EFFECTS. 1987-2023

Independent variable (t student)	Baseline	Fixed effects	Random effects
η Technology	1.01 (5.13)***		1.01 (5.13)***
Durable goods		1.018 (5.47)***	1.04 (9.48)***
Nondurable goods		1.007 (1.93)***	0.98 (0.70)
β Labor	0.33 (46.54)***	0.33 (46.54)***	
Durable goods			0.40 (76.06)***
Nondurable goods			0.24 (28.47)***
α Capital	0.20 (36.86)***	0.20 (36.86)***	
Durable goods			0.17 (215.24)***
Nondurable goods			0.23 (29.13)***
γ Intermediate inputs	0.47 (81.12)***	0.47 (81.12)***	
Durable goods			0.42 (-3.75)***
Nondurable goods			0.51 (63.37)***
RMSE	0.0618	0.0616	0.0301

Notes: number of observations is 703. The 336 industry is divided in two parts by the OPT. The Stata 18 program was used.

Considering the without effects column, the technology for the whole manufacturing in the United States presents increasing returns to scale, although in a marginal manner, i.e., 1.01. At a national manufacturing sector level, labor takes 33% of the value added, capital the 20%, and intermediate inputs the 47%. In Carbajal-De-Nova (2024) labor and capital shares on value added were 11% and 89% respectively for all sectors and industries in the United States. It is possible, in this last case, that capital have included intermediate inputs in the sense that the OPT defines. If this were true, then the capital share is 20%+47%=67%, while labor share is 0.33. Income distribution favors capital during this period, in line with Piketty (2015), Piketty (2021) research. Elasticities are given in this description in percentage terms.

Regarding individual fixed effects, between durable and nondurable goods the most efficient sector is the first one: 101.8% versus 100.7%. The shares of labor, capital and intermediate inputs remain unchanged for the manufacturing sector with respect to the without effects column. Labor share is larger in durables (41%), than in nondurables (25%). Conversely, the capital share is larger in nondurable (23%), than in durable goods (175). Similarly, the share of intermediate inputs is larger in nondurable (51%), than in durable goods (42%).

Table 3 presents the results of a panel model without effects at the three-digit industry granularity level, where ROLS methodology is fully applied. Among the three-digit industries, the most efficient industry in manufacturing is 3364-9 other transportation equipment, with $\eta = 1.02$ or 102%, indicating increasing returns to scale. The most labor intensive industry is 334 computer and electronic products with $\alpha = 0.60$, or with labor share in total value added of 60%. The most capital intensive industry is 325 chemical products with $\beta = 0.33$, or with a capital share in total value added of 33%. 3361-3 Motor vehicles, bodies and trailers, and parts is the industry that uses the most intermediate inputs with a $\gamma = 0.78$, or 78% share in total value added. Further analysis that considers the trade balance for manufacturing exports and imports would clarify what proportion of this 78% intermediate input share is composed by exports and imports.

Table 2 and 3 include all production factors, as OPT data ensures that the sum of capital costs, labor costs, and intermediate inputs cost is equal to sectoral output. Also, OPT assures that sectoral output converges with value added, as previously mentioned. Although both Table 2 and 3 do no display full estimates decimal precision, their estimates values sum up to one, consisted with ROLS methodology. The η elasticity of technology exhibits coefficients different to one, therefore, it seems that there is not restriction that $\eta = 1$ as Samuelson (1979) once argued. This author argues that $\eta = 1$ would always hold, letting the Cobb-Douglas production function estimations unable to express technical progress.

Chirinko (2002) and Berndt (1976) note that cross-sectional studies at the two-digit level tended to find elasticities insignificantly different from one. They assume that technological change is Hicks neutral. Technological efficiency is Hicks neutral if it does not influence the ratio of marginal products, for a given capital-labor ratio. For their part, Dhrymes and Zarembka (1970) use two assumptions to compute elasticities of substitution for two-digit United States manufacturing industries. These assumptions fall within perfect competition outlined in Arrow et. al., (1961), as well as with a Cobb-Douglas production function homogeneous of degree one. The data use by these authors included value added, wage bill, number of employees, and the net book value of the capital stock for a given industry in each state. Their results indicated that final consumption-oriented industries have high elasticities of substitution. On the other hand, investment-oriented industries tend to be characterized by relatively low elasticities of substitution. They conclude that a constant elasticity of substitution production function does not uniformly describe well the production process of the two-digit industries analyzed. So, for these authors Hicks neutral technological efficiency is not observable.

TABLE 3
RESULTS OF THE ESTIMATES OF EQUATION (3a). UNITED STATES MANUFACTURING: THREE-DIGIT MANUFACTURING INDUSTRIES: FIXED AND RANDOM EFFECTS. 1987-2023

Naics code	Industry	η Technology	α Labor	β Capital	γ Intermediate inputs	RMSE
311-312	Food and beverage and tobacco products	0.99 (37.68)***	0.14 (76.06)***	0.15 (215.24)***	0.70 (-3.75)***	0.0016
313-314	Textile mills and textile product mills	1.003 (3.89)***	0.28 (46.43)***	0.075 (22.46)***	0.075 (113.06)***	0.0041

315-316	Apparel and leather and applied products	0.96 (4.27)***	0.32 (17.99)***	0.23 (13.63)***	0.43 (55.23)***	0.0330
321	Wood products	1.01 (12.78)***	0.29 (21.64)***	0.11 (25.14)***	0.58 (2.95)***	0.0134
322	Paper products	0.99 (1.77)*	0.23 (82.06)***	0.18 (64.75)***	0.57 (209.49)***	0.0026
323	Printing and related support activities	1.001 (0.70)*	0.30 (17.29)***	0.10 (42.65)***	0.58 (35.86)***	0.0060
324	Petroleum and coal products	0.96 (6.95)***	0.05 (12.59)***	0.20 (36.11)***	0.74 (108.92)***	0.0106
325	Chemical products	0.987 (4.46)***	0.17 (18.78)***	**0.33 (57.05)*****	0.49 (54.64)***	0.0091
326	Plastics and rubber products	0.99 (1.38)*	0.23 (58.59)***	0.12 (39.85)***	0.64 (214.12)***	0.0025
327	Nonmetallic mineral products	0.99 (1.82)**	0.28 (32.02)***	0.18 (48.33)***	0.52 (48.80)***	0.0051
331	Primary metal products	0.99 (3.45)***	0.19 (34.13)***	0.10 (15.85)***	0.70 (61.65)***	0.0062
332	Fabricated metal products	0.99 (2.81)***	0.32 (99.61)***	0.12 (35.68)***	0.54 (202.12)***	0.0020
333	Machinery	0.99 (3.06)***	0.30 (100.46)***	0.15 (52.81)***	0.54 (147.92)***	0.0032
334	Computer and electronic products	1.01 (2.31)***	**0.60** (26.74)***	0.10 (5.59)***	0.28 (41.22)***	0.0280
335	Electrical equipment, appliances, and components	1.003 (1.21)	0.30 (29.98)***	0.17 (31.39)***	0.52 (74.91)***	0.0057
3361-3	Motor vehicles, bodies and trailers, and parts	0.99 (0.34)	0.17 (17.17)***	0.04 (12.12)***	**0.78 (74.30)*****	0.0149
3364-9	Other transportation equipment	**1.02 (6.08)*****	0.40 (22.43)***	0.17 (22.62)***	0.42 (29.71)***	0.0172
337	Furniture and related products	0.99 (0.17)	0.32 (103.75)***	0.08 (41.86)***	0.58 (204.27)***	0.0020

Notes: number of observations is 703. NAICS stands for North America Industrial Classification System. The 336 industry is divided in two parts by the OPT. The Stata 18 program was used.

CONCLUSION

There have been many attempts in the literature to measure economic activity and its underlying relationships. Many authors who have sought to empirically estimate the Cobb-Douglas production function have faced challenges related to availability data and its alignment with the theorical framework, as well as to the assumptions embedded in the production function. Frequently, researchers must construct indexes from available data to estimate technology change, as well as labor and capital shares.

However, due to increased data availability, advances in computational science, and a growing number of available Input-Output matrices, National Statistics Offices are now capable of producing sectoral output indicators, that account for intermediate inputs costs (excluding intra-industry transactions). The inclusion of intermediate inputs ensures that sectoral output is fully allocated to production factors payments. In contrast to earlier studies, which identified a residual typically attributed to missing production factors, this note finds no such a residual. All components of sectoral output or value added are explicitly accounted for. In Carbajal-De-Nova (2024) labor and capital shares on value added were reported ad 11% and 89%, respectively, across all United States sectors and industries. It is possible, that in this case capital shares included intermediate inputs, as defined by the OPT.

According to the results in Tables 2 and 3, the elasticity of technology (η), or total factor productivity deviates from one. Sectoral output or value added, as defined by OPT, consist of the sum of labor, capital and intermediate costs. Following Nerlove (1965), the corresponding cost function is an appropriate reduce-form equation to represent production relationships. Therefore, the constraint $\alpha + \beta + \gamma = 1$, applied through ROLS estimation, holds invariant with respect to input shares sum, but not to output technology growth. This explains the variation in technology measures among manufacturing industries, even under ROLS constraint. It appears that Samuelson (1979) criticism, that Cobb-Douglas production functions do not exhibit technical progress would not hold true.

The elasticity coefficients on Table 2 suggest that the manufacturing sector have a marginal increasing returns to scale, i.e., 1.01, labor share is 0.33, capital share is 0.20, and intermediate inputs share is 0.47. If intermediate inputs are "fixed" inputs as defined by the OPT, then the total manufacturing capital share would be 0.20 plus 0.47 equals to 0.67, while the manufacturing labor share is 0.33. Therefore, income distribution favors capital during this period. Technology in durable goods is more efficient than in nondurable goods (1.018 vs 1.007). For durable goods, labor, capital and intermediate shares are 0.40, 0.17, and 0.42, respectively. For nondurable goods, labor, capital and intermediate shares are 0.24, 0.23, and 0.51, respectively. At the three-digit industry level, the most efficient industry is 3364-9 (other transportation equipment), which exhibits increasing returns to scale with $\eta = 1.02$. The most labor-intensive industry is 334 (computer and electronic products), with a share of 0.60. The most capital-intensive industry is 325 (chemical products) with a share of 0.33, and the most intermediate input-intensive industry is 3361-3 (motor vehicles, bodies and trailers), with a share of 0.78. Future work could incorporate the trade balance to clarify what proportion of the 0.78 intermediate input share is accounted for exports and imports.

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